Journal of Sound and Vibration (1999) **219**(4), 723–729
Article No. jsvi.1998.1849, available online at http://www.idealibrary.com.on

LETTERS TO THE EDITOR

THE INFLUENCE OF THE ELECTROMAGNETIC BALANCING REGULATOR ON THE ROTOR SYSTEM*

S. Zeng and X.-X. Wang

Institute of Chemical Machinery, Zhejiang University, Hangzhou 310027, *P.R.C.*

(*Received* 23 *December* 1997, *and in final form* 29 *June* 1998)

1. INTRODUCTION

The electromagnetic balancing regulator (EBR) is very special, for it makes of non-contact electromagnetic force to drive the correction masses so as to generate suitable correction weight. Its working principle, as well as the experiment on a model fan with a rigid rotor, have been introduced in reference [1], and the experimental result is satisfied. Furthermore, in reference [2] the application of the EBR to the flexible rotor has also been studied, and an on-line automatic balancing system with a flexible rotor is built. The experiment on it shows that the EBR can efficiently reduce the vibration of the flexible rotor.

The EBR is a non-linear operator. During its working period, the circumferential force and the radial force are probably generated simultaneously. So it may influence the stability of the flexible rotor. Here the influence of the EBR on the stability of the flexible rotor is studied. When the regulator is started, the simulated instantaneous response of the rotor is also involved.

2. RADIAL ELECTROMAGNETIC FORCE

The electromagnetic force is the working basis of the EBR [1, 2]. When the needed circumferential force is generated, the radial electromagnetic force will be produced with the existence of the deviation caused by unbalance. The radial force direction of the EBR is the same as that of the deviation, and the magnitude is governed [3] by

$$F = \mu_0 A N^2 I^2 \left(\frac{2x}{\delta_0^3} + \frac{3x^3}{\delta_0^5} \right) \tag{1}$$

where A = cross-section area of a pole, μ_0 = permeability of the vacuum, N = turn number of the coil, I = electric current fed in coil, δ_0 = equivalence gap between stator and rotor, and x = deviation;

3. MOTION EQUATION OF ROTOR SYSTEM WITH EBR

For an easy understanding, on the basis of the test rig in reference [2], a JEFFCOTT rotor with an EBR and a working disk is constructed, in which the span and the diameter of the shaft are 700 mm and 24 mm respectively, as shown

* This paper was sponsored by the National Natural Science Foundation of China.

0022–460X/99/040723 + 07 $30.00/0

in Figure 1. The weight of the disk is 5 kg, which is the reduced value of all disks in reference [2]. The stiffness at the position of the working disk is calculated to be $k = 5 \times 10^5$ N/m.

The JEFFCOTT rotor may be regarded as a single freedom vibration system. If the EBR is off, the situation is a general one, which will not be mentioned here. When the EBR is on, the motion equation without damping will be

$$m\ddot{x} + kx - \mu_0 AN^2I^2\left(\frac{2x}{\delta_0^3} + \frac{3x^3}{\delta_0^5}\right) = 0 \tag{2}$$

or

$$\ddot{x} + \omega_0'^2(x + \beta x^3) = 0 \tag{3}$$

where

$$\omega_0 = \sqrt{k/m}$$

is the inherent frequency of the rotor system when the EBR is off;

$$\omega_0' = \sqrt{\frac{k - 2\mu_0 AN^2I^2/\delta_0^3}{m}}; \qquad \beta = -\frac{(3\mu_0 AN^2I^2/\delta_0^5)}{\omega_0'^2}.$$

From equation (3) it is found that when the EBR is on, the inherent frequency will reduce because of the stiffness decrease.

In equation (3), in view of $\beta < 0$, let $-\beta = \rho^2$, then the singular points of the conservation system represented by equation (3) are (0, 0), $(1/\rho, 0)$ and $(-1/\rho, 0)$. According to the singular point theory [4], in the phase plane the phase tracks near the point (0, 0) are all enclosed ones, and with the decrease of the whole system potential energy, they converge to the stable center (0, 0). This indicates that this time the vibration energy of the rotor system is weak, and the maximum displacement of the rotor does not exceed $1/\rho$, which is the normal working state of the rotor. In comparison, the phase tracks near the point $(1/\rho, 0)$ and point $(-1/\rho, 0)$ are not enclosed ones. This indicates that the vibration energy of the rotor system increases to such an extent that the displacement of the rotor exceeds $1/\rho$, which will make the vibration energy and the displacement increase further and the vibration diverge. This is an unstable balancing status, which will lead to

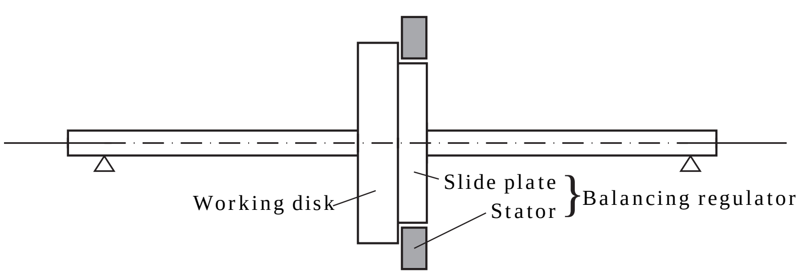

Figure 1. JEFFCOTT rotor.

the rotor destabilizing and the slide plate colliding with the stator. For this JEFFCOTT rotor, $1/\rho = 333$ μm.

4. THE INSTANTANEOUS RESPONSE OF THE ROTOR SYSTEM WHEN THE EBR IS STARTED

The instantaneous response in this section is subjected to a situation in which the constructed JEFFCOTT rotor operates at the vicinity of the stable singular point (0, 0), i.e. vibration displacement of the rotor is much smaller than $1/\rho$, or 333 μm.

Before the EBR is started, the response of the rotor system is a harmonious forced vibration, which should be the initial condition of the instantaneous response. Hence it must be determined first. If the mass-radius product of the unbalance emerges in the working disk is U, then the motion equation of the JEFFCOTT rotor with damping in a transverse direction is

$$m\ddot{x} + c\dot{x} + kx = U\omega^2 \cos(\omega t) \tag{4}$$

and the stable state solution is

$$x = \frac{U\omega^2 \cos(\omega t - \varphi)}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \qquad \varphi = tg^{-1}\frac{c\omega}{k - m\omega^2}. \tag{5}$$

It is assumed that when the maximum displacement of the rotor occurs, the EBR is started. Then the instantaneous response of the rotor with EBR should be governed by

$$m\ddot{x} + c\dot{x} + kx - \mu_0 A N^2 I^2\left(\frac{2x}{\delta_0^3} + \frac{3x^3}{\delta_0^5}\right) = U\omega^2\cos(\omega t). \tag{6}$$

The initial condition is

$$\begin{cases} x|_{t=0} = \dfrac{U\omega^2}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \\ \dot{x}|_{t=0} = 0. \end{cases} \tag{7}$$

Equation (6) can be rewritten as follows

$$\ddot{x} + \omega_0'^2 x = \varepsilon f(\omega t, x, \dot{x}) \tag{8}$$

where

$$x = \text{the displacement of the rotor;}$$

$$f(\omega t, x, \dot{x}) = F\cos(\omega t) - 2\sigma\omega_0'\dot{x} - \omega_0'^2 x^3;$$

$$\omega_0' = \sqrt{\frac{k - 2\mu_0 A N^2 I^2/\delta_0^3}{m}};$$

$$\varepsilon = -\frac{3\mu_0 A N^2 I^2}{\delta_0^5 \omega_0'^2} \times 10^{-12} \text{ (small parameter)}$$

$$\sigma = \frac{\xi\sqrt{km}}{m\varepsilon\omega_0'};$$

$$\xi = \frac{c}{2\sqrt{km}} \text{ (damping ratio of the rotor system)};$$

$$F = \frac{U\omega^2}{m\varepsilon} \times 10^6.$$

Equation (8) is a non-linear one, and may be solved with the asymptotic method [4]. The basic rationale of this method is to seek the series solution with asymptotic property according to the approximate harmony character of the vibration of the weak non-linear system. In the light of asymptotic method, the solution of equation (8) is

$$x = a\cos\varphi + \frac{1}{32}\varepsilon a^3 \cos 3\varphi + \frac{\varepsilon F}{\omega_0'^2 - \omega^2}\cos\omega t$$

$$a = a_0 \mathrm{e}^{-\varepsilon\sigma\omega'_0 t} \tag{9}$$

$$\varphi = \omega'_0 t - \frac{3}{16}\frac{1}{\sigma} a_0^2 (\mathrm{e}^{-2\varepsilon\sigma\omega'_0 t} - 1) + \varphi_0$$

where a_0 and φ_0 are determined by the initial condition (14).

In the instantaneous response represented by equation (9), there are free vibration term $a\cos\varphi$, forced vibration term $\varepsilon F/(\omega_0'^2 - \omega^2)\cos\omega t$ and triplex basic frequency term $(1/32)\varepsilon a^3 \cos 3\varphi$. Other terms are neglected for they are too small comparatively. From equation (9), it is noted that if there is no damping in the rotor system, none of the terms will decline; but, if there is, so long as the declining time is long enough, all the parts except the forced vibration will vanish gradually.

For the JEFFCOTT rotor in this paper, it is suggested that the rotating speed by 2700 rpm, and the mass-radius product of the unbalance mr in rotor to be $1{\cdot}252 \times 10^{-4}$ kg.m, which will produce 10 N unbalance exciting force. The instantaneous response curves solved from equations (7) and (9) with a damping ratio ξ from 0 to 0·02 are shown in Figure 2. From Figure 2 it is noted that when the EBR is off, the rotor keeps a stable harmonic vibration. As the dampings in the rotor system are relatively small, and the amplitudes of the harmonic vibration are very close to each other with a value of 98 μm, which means that the initial displacement of the instantaneous is 98 μm.

In Figure 2, it is found that the rotor system is disturbed at the EBR's start. And whether there is damping or not, the vibration amplitude will increase, which is harmful to the rotor system (especially, when the amplitude exceeds 333 μm determined by the unstable singular points ($\pm 1/\rho$,0), the rotor may be out of stability, which must be avoided in design and practice). If there is no damping, the vibration of the rotor will maintain all the time, the free vibration part will not decline, and the amplitude will be always much bigger than the initial one. On the contrary, while there is damping in the rotor system, the free vibration term will weaken gradually, and the bigger the damping ratio is, the faster the vibration

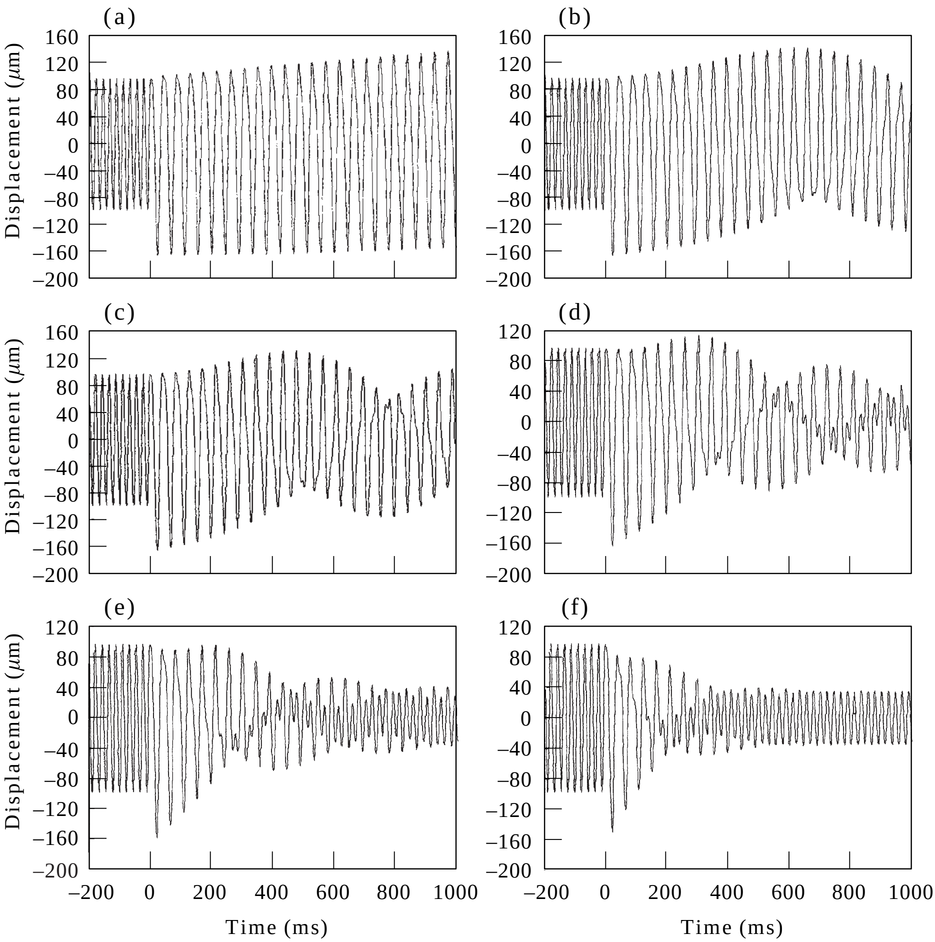

Figure 2. The instantaneous responses with different damping ratios. ξ = (a) 0; (b) 0·001; (c) 0·002; (d) 0·005; (e) 0·01; (f) 0·02.

will attenuate. Therefore there must be suitable damping in the rotor system to decrease the instantaneous free vibration amplitude which is caused by the EBR and may be quite considerable.

From Figure 2 it is also noted that the rotor system will retain a low vibration level if there is damping in the system and the operating time of the EBR is long enough. This condition is due to the stiffness reduction of the rotor at the EBR's start. For the JEFFCOTT rotor in this paper, the stiffness reduction causes the working rotating to be further from the critical rotating speed. As a result, the vibration amplitude reduces, which is shown in Figure 3. In Figure 3 when the EBR is not started, the BODE curve of the rotor is curve A, whose critical rotating speed is 3000 rpm, the working rotating speed is 2700 rpm, so the working point is Q. When the EBR is started, the BODE curve is changed to curve B with a critical rotating speed of about 1442 rpm, but the working rotating speed is still 2700 rpm. The working point hence drifts to H, and the vibration of point H is obviously smaller than that of point Q. The damping ratio in Figure 3 is $\xi = 0{\cdot}02$.

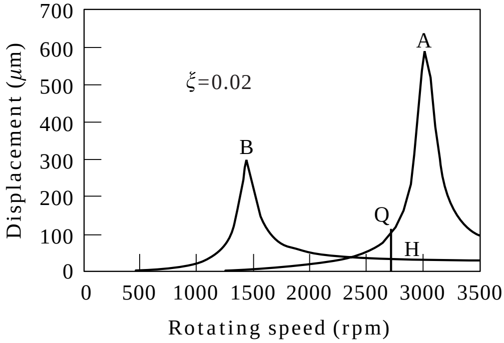

Figure 3. BODE figure of the JEFFCOTT rotor system.

The condition in Figure 3 is a good one, in which, when the EBR is on, the working frequency becomes further from the (reduced) inherent frequency, and the amplitude of the forced vibration is lower than that without the EBR. But one is reminded that another condition should be avoided, in which working frequency is at the vicinity of the (reduced) inherent frequency otherwise, the amplitude of the rotor may be very considerable. For example, in Figure 3 if the working rotating speed is about 1450 rpm when the EBR is started, the vibration amplitude will increase dramatically.

Lastly it must be pointed out that the working point's drifting happens only when the unbalance in the machine is so considerable that the EBR has to be started to balance it. During the periods of the machine being switched on or off, the EBR stops working and, so has no influence on the rotor.

5. CONCLUSION

In this paper, having studied the influence of the EBR on the stability of the rotor system and the instantaneous response at the start of the EBR, the following conclusions can be drawn.

(1) After EBR is installed on the rotor system, the inherent frequency of the rotor is reduced because of the extra attachment mass of the EBR. Besides that, when the EBR is in operation, the stiffness of the rotor will decrease further. As a result, the inherent frequency will drop even more. So it is noted that the rotor should not work at the vicinity of the rotating speed corresponding to the dropped inherent frequency.

(2) During the operation of the EBR, the rotor system is in a state of local stability. In its phase plane there are two unstable singular points, and at no time should the displacement of the rotor exceed that corresponding to the singular point otherwise the rotor will destabilize. The displacement will increase dramatically, and the rotor will collide with the stator.

(3) At the moment that the EBR is started, the rotor is disturbed and the rotor displacement increases. If there is damping in the rotor system, the increased rotor displacement will decrease. And the greater the damping is, the faster the displacement decreases.

REFERENCES

1. Sheng Zeng and Xi-xuan Wang 1998 *Journal of Sound and Vibration* **209**(1), 5–13. The electromagnetic balancing regulator and the automatic balancing system.
2. Xi-xuan Wang and Sheng Zeng 1997 *Proceeding of DECTC*'97, 1997 *ASME Design Engineering Technical Conference*, *Sacramento*, *Calif*. A study on electro-magnetic auto-balancing system and its balancing method.
3. M. E. F. Kasarda and P. E. Allaire 1990 *Journal of Engineering for Gas Turbines and Power* **112**, 463–469. A magnetic damper for first mode vibration reduction in multimass flexible rotor.
4. Wen-mei Ji 1985 *Mechanical Vibration*. Beijing: Science Press (in Chinese).